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基于积分型 Lyapunov 函数的随机非线性系统的自适应控制

王 飞 张天平✉ 施枭铖

扬州大学信息工程学院自动化专业部，扬州 225127

✉ 通信作者，E-mail: tpzhang@ yzu. edu. cn

摘 要 针对一类带有未知虚拟控制增益的随机严格反馈非线性系统，基于后推设计，引入积分型 Lyapunov 函数，并利用神经网络的逼近能力，提出了一种自适应神经网络控制方案. 与现有研究结果相比，放宽了对控制系统的要求，取消了对于未知函数的限制条件. 通过 Lyapunov 方法证明了闭环系统的所有误差信号依概率有界. 仿真结果验证了所给控制方案的有效性.

关键词 非线性系统; 随机系统; 自适应控制; 神经网络; 后推

分类号 TP273⁺.2

Adaptive control for stochastic nonlinear systems based on the integral-type Lyapunov function

WANG Fei, *ZHANG Tian-ping*✉, *SHI Xiao-cheng*

Department of Automation, College of Information Engineering, Yangzhou University, Yangzhou 225127, China

✉ Corresponding author, E-mail: tpzhang@ yzu. edu. cn

ABSTRACT Based on the backstepping technique, introducing the integral-type Lyapunov function and utilizing the approximation capability of neural networks, an adaptive neural network control scheme was proposed for a class of stochastic strict-feedback nonlinear systems with unknown virtual control gain. Compared with existing literatures, the proposed approach relaxes the requirements of the control system and cancels the restriction of the unknown function. By the Lyapunov method, it is shown that all error variables in the closed-loop system are bounded in probability. Simulation results illustrate the effectiveness of the proposed control scheme.

KEY WORDS nonlinear systems; stochastic systems; adaptive control; neural networks; backstepping

近年来，随机非线性系统已成为控制理论研究的热点之一，如何将确定性系统的控制技术推广到随机非线性系统已经成为一个公开的研究领域，并取得了一些研究成果[1-8]. 随机系统 Lyapunov 函数设计的主要技术障碍在于伊藤随机微分不仅涉及梯度项还涉及高阶 Hessen 矩阵项. 文献[1]应用后推方法，首次解决了一类下三角型结构的随机非线性系统的镇定控制器设计问题. 文献[4-5]首次提出采用四次型 Lyapunov 函数取代经典的二次函数进行后推设计，并研究了输出反馈镇定问题. 文献[9-10]研究了一类随机严格反馈非线性系统的自适应神经网络输出反馈镇定. 文献[11]研究了一类不确定随机非线性时滞系统的自适应有界镇定问题，但是其虚拟控制及控制律结构相对复杂. 文献[12]提出一种自适应神经网络控制方案来解决未知协方差噪声干扰的不确定非线性系统的输出跟踪控制问题. 文献[13]解决了确定性系统中未知控制增益带来的问题，文献[14]针对一类带有未知虚拟控制增益的随机非线性系统，提出了一种模糊自适应控制方法. 但是其对未知函数条件要求比较严格.

本文基于后推设计方法和积分型 Lyapunov 函数，并利用神经网络的逼近能力，提出一种自适应控制方案，利用 Lyapunov 方法，证明闭环系统依概率渐近稳定，跟踪误差收敛到一个小的残差集内. 与已有文献相比本文所研究的系统更一般，取消了

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文献[15]对于未知函数在零点等于零的条件限制.

1 问题描述及基本假设

考虑如下随机非线性严格反馈系统:

$$\begin{cases} \mathrm{d}x_i = (f_i(\bar{x}_i) + g_i(\bar{x}_i)x_{i+1})\mathrm{d}t + \\ \qquad h_i^{\mathrm{T}}(\bar{x}_i)\mathrm{d}\boldsymbol{w}, i = 1,2,\cdots,n-1; \\ \mathrm{d}x_n = (f_n(\bar{x}_n) + g_n(\bar{x}_n)u)\mathrm{d}t + h_n^{\mathrm{T}}(\bar{x}_n)\mathrm{d}\boldsymbol{w}; \\ y = x_1. \end{cases} \quad (1)$$

式中:$x_i \in \mathbf{R}(i = 1,2,\cdots,n)$为系统的状态,并定义$\bar{\boldsymbol{x}}_i = (x_1,x_2,\cdots,x_i)^{\mathrm{T}}$;$u \in \mathbf{R}$为控制输入;$y$为系统输出;$f_1(\bar{x}_1),f_2(\bar{x}_2),\cdots,f_n(\bar{x}_n),g_1(\bar{x}_1),g_2(\bar{x}_2),\cdots,g_n(\bar{x}_n),h_1(\bar{x}_1),h_2(\bar{x}_2),\cdots,h_n(\bar{x}_n)$都是未知连续函数;$w$是定义在完备概率空间$(\Omega,F,P)$上的$r$维标准布朗运动,其中$\Omega$为样本空间,$F$为$\sigma$代数,$P$为概率测度. 本文的主要目的是设计一个自适应状态反馈控制律u,要求系统输出y尽可能的跟踪一个指定的期望轨迹y_{d},使得闭环系统依概率渐近稳定,跟踪误差依概率收敛到一个小的残差集内.

对于具有以下形式的系统:

$$\mathrm{d}\boldsymbol{x} = f(\boldsymbol{x},t)\mathrm{d}t + \boldsymbol{h}^{\mathrm{T}}(\boldsymbol{x},t)\mathrm{d}\boldsymbol{w}. \quad (2)$$

式中,x和w的定义与式(1)相同,而$f: \mathbf{R}^n \times \mathbf{R}^+ \to \mathbf{R}^n$,$\boldsymbol{h}^{\mathrm{T}}: \mathbf{R}^n \times \mathbf{R}^+ \to \mathbf{R}^{n\times r}$对$t$一致地关于变量$x$满足局部Lipschitz条件,且$f(t,0)$和$\boldsymbol{h}(t,0)$关于$t$一致有界. 对二次可微函数$V(t,\boldsymbol{x}(t))$定义如下微分算子:

$$\mathrm{L}V(t,\boldsymbol{x}(t)) = \frac{\partial V(t,\boldsymbol{x}(t))}{\partial t} + \frac{\partial V(t,\boldsymbol{x}(t))}{\partial \boldsymbol{x}^{\mathrm{T}}}f + \frac{1}{2}\mathrm{tr}\left\{\boldsymbol{h}\frac{\partial^2 V(t,\boldsymbol{x}(t))}{\partial \boldsymbol{x}^{\mathrm{T}}\partial \boldsymbol{x}}h^{\mathrm{T}}\right\}. \quad (3)$$

式中,$\mathrm{tr}(A)$为A的迹.

定义1 如果对任意$\varepsilon: 0 < \varepsilon < 1$,对所有初值$x_0 \in S_0$($S_0$为某个包含原点的紧集),存在紧集$S \supset S_0$,有$\inf\limits_{x_0 \in S_0} P\{x(t,t_0,x_0) \in S, \forall t \geq t_0\} \geq 1 - \varepsilon$,则称随机系统(2)的解$x(t)$为半全局依概率有界.

引理1[6,15] 对于系统(2),如果存在二次可微函数$V(t,\boldsymbol{x}(t))$,$\mu_1,\mu_2 \in \kappa_\infty$($\kappa_\infty$表示所有连续,严格递增,在零点等于零的$\mathbf{R}^+ \to \mathbf{R}^+$的无界函数集合),常数$c_1 > 0, c_2 \geq 0$,使得

$$\mu_1(|\boldsymbol{x}|) \leq V(t,\boldsymbol{x}) \leq \mu_2(|\boldsymbol{x}|),$$
$$\mathrm{L}V \leq -c_1V + c_2,$$

则系统(2)在$(t_0,+\infty)$上存在唯一解,且解过程是依概率有界的.

假设1 存在正常数g_{i0}和g_{i1},使得$0 < g_{i0} \leq g_i(\bar{x}_i) \leq g_{i1}$,$\forall \bar{\boldsymbol{x}}_i \in \mathbf{R}^i, i = 1,2,\cdots,n$,且$g_i(\bar{x}_i)(\forall t \geq 0)$是二次可微的.

假设2 期望的轨迹向量$\boldsymbol{x}_{i\mathrm{d}}$是连续的,且$\boldsymbol{x}_{i\mathrm{d}} \in \Omega_{\boldsymbol{x}_{i\mathrm{d}}} \subset \mathbf{R}^{i+1}, i = 1,2,\cdots,n$. 其中$\boldsymbol{x}_{i\mathrm{d}} = (y_{\mathrm{d}},\dot{y}_{\mathrm{d}},\cdots,y_{\mathrm{d}}^{(i)})^{\mathrm{T}}$,$\Omega_{\boldsymbol{x}_{i\mathrm{d}}}$是已知的有界闭集.

本文中,未知光滑非线性函数$\psi(x): \mathbf{R}^n \to \mathbf{R}$将在紧集$\Omega_x$上采用如下径向基函数神经网络进行逼近:

$$\psi(\boldsymbol{x}) = \boldsymbol{W}^{\mathrm{T}}\boldsymbol{\varphi}(x) + \varepsilon(\boldsymbol{x}), \ \forall \boldsymbol{x} \in \Omega_x. \quad (4)$$

式中:逼近误差$|\varepsilon(x)| \leq \varepsilon$,$\varepsilon$为未知正常数;$\boldsymbol{\varphi}(\boldsymbol{x}) = (\varphi_1(\boldsymbol{x}),\cdots,\varphi_l(\boldsymbol{x}))^{\mathrm{T}}: \Omega_x \to \mathbf{R}^l$是已知光滑向量函数,神经网络节点数$l > 1$,基函数$\varphi_i(\boldsymbol{x}), 1 \leq i \leq l$取作通常形式的高斯函数,即

$$\varphi_i(\boldsymbol{x}) = \exp\left(\frac{-\|\boldsymbol{x} - \mu_i\|^2}{\eta_i^2}\right), \ i = 1,2,\cdots,l.$$

式中,μ_i、η_i分别是高斯函数的中心和宽度. 理想权向量$\boldsymbol{W} = (w_1,w_2,\cdots,w_l)^{\mathrm{T}}$定义为

$$\boldsymbol{W} = \arg\min_{\hat{W} \in \mathbf{R}^l}\{\sup_{x \in \Omega_x}|\psi(\boldsymbol{x}) - \hat{\boldsymbol{W}}^{\mathrm{T}}\boldsymbol{\varphi}(\boldsymbol{x})|\}.$$

2 自适应神经网络控制器设计

采用后推方法设计自适应状态反馈控制器,其步骤如下,定义如下坐标变换:

$$\begin{cases} z_i = x_i - \alpha_{i-1}, i = 1,2,\cdots,n; \\ \alpha_0 = y_{\mathrm{d}}. \end{cases} \quad (5)$$

第一步($i = 1$),由系统(1)及式(5)可得

$$\mathrm{d}z_1 = \mathrm{d}x_1 - \mathrm{d}y_{\mathrm{d}} = (f_1(\bar{x}_1) + g_1(\bar{x}_1)x_2 - \dot{y}_{\mathrm{d}})\mathrm{d}t + \boldsymbol{h}_1^{\mathrm{T}}(\bar{x}_1)\mathrm{d}\boldsymbol{w}. \quad (6)$$

定义积分型Lyapunov函数

$$V_{z_1} = \int_0^{z_1}\frac{\sigma^3}{g_1(\sigma + y_{\mathrm{d}})}\mathrm{d}\sigma. \quad (7)$$

由积分中值定理知,$\exists \lambda_1 \in (0,1)$,使得$V_{z_1} = \frac{z_1^4}{4g_1(\lambda_1 z_1 + y_{\mathrm{d}})}$,因为$0 < g_{10} \leq g_1(\bar{x}_1) \leq g_{11}$,所以$\frac{z_1^4}{4g_{11}} \leq V_{z_1} \leq \frac{z_1^4}{4g_{10}}$,故$V_{z_1}$是关于变量$z_1$的正定函数,由式(3)可得

$$\mathrm{L}V_{z_1} = \frac{z_1^3}{g_1(\bar{x}_1)}[f_1(\bar{x}_1) + g_1(\bar{x}_1)(z_2 + \alpha_1) - \dot{y}_{\mathrm{d}}] + \dot{y}_{\mathrm{d}}\left[\frac{z_1^3}{g_1(\bar{x}_1)} - z_1^3\int_0^1\frac{3\rho_1^2}{g_1(\rho_1 z_1 + y_{\mathrm{d}})}\mathrm{d}\rho_1\right] + \frac{3z_1^2 g_1^{-1}(\bar{x}_1) + z_1^3\frac{\partial g_1^{-1}(\bar{x}_1)}{\partial z_1}}{2}\boldsymbol{h}_1^{\mathrm{T}}(\bar{x}_1)\boldsymbol{h}_1(\bar{x}_1). \quad (8)$$

由Young's不等式

$$\frac{3z_1^2 g_1^{-1}(\bar{x}_1)}{2}\boldsymbol{h}^{\mathrm{T}}(\bar{x}_1)\boldsymbol{h}(\bar{x}_1) \leq$$

$$\frac{1}{\eta_1}+\frac{9\eta_1}{16g_1^2(\bar{\boldsymbol{x}}_1)}\|\boldsymbol{h}_1(\bar{\boldsymbol{x}}_1)\|^4 z_1^4, \tag{9}$$

$$z_1^3 z_2 \leqslant \frac{3}{4}\lambda_1^{\frac{4}{3}} z_1^4+\frac{1}{4\lambda_1^4} z_2^4, \tag{10}$$

将式(9)、式(10)代入式(8)得

$$\mathrm{L}V_{z_1}\leqslant \frac{1}{\eta_1}+z_1^3(\alpha_1+\psi_1(\bar{z}_1))+\frac{1}{4\lambda_1^4}z_2^4. \tag{11}$$

式中，

$$\begin{aligned}\psi_1(\bar{z}_1)=&\frac{f_1(\bar{\boldsymbol{x}}_1)}{g_1(\bar{\boldsymbol{x}}_1)}+\frac{3}{4}\lambda_1^{\frac{4}{3}}z_1-\dot{y}_{\mathrm{d}}\int_0^1\frac{3\rho_1^2}{g_1(\rho_1 z_1+y_{\mathrm{d}})}\mathrm{d}\rho_1+\\&\frac{9\eta}{16g_1^2(\bar{\boldsymbol{x}}_1)}\|\boldsymbol{h}_1(\bar{\boldsymbol{x}}_1)\|^4 z_1+\frac{1}{2}\frac{\partial g_1^{-1}(\bar{\boldsymbol{x}}_1)}{\partial z_1}\|\boldsymbol{h}_1(\bar{\boldsymbol{x}}_1)\|^2,\\&\bar{z}_1=(x_1,y_{\mathrm{d}},\dot{y}_{\mathrm{d}})\in\mathbf{R}^3.\end{aligned}$$

取虚拟控制律

$$\alpha_1=-k_1 z_1-\hat{\psi}_1(\bar{z}_1) \tag{12}$$

式中，$k_1>0$ 是设计常数，$\hat{\psi}_1(\bar{z}_1)=\hat{\boldsymbol{W}}_1^{\mathrm{T}}\varphi_1(\bar{z}_1)$ 是径向基函数神经网络在有界闭集 $\Omega_{\bar{z}_1}$ 上对 $\psi_1(\bar{z}_1)$ 的逼近，$\hat{\boldsymbol{W}}_1$ 是理想权向量 W_1 的估计，估计误差定义为 $\tilde{\boldsymbol{W}}_1=\hat{\boldsymbol{W}}_1-\boldsymbol{W}_1$. 令 $\psi_1(\bar{z}_1)=\boldsymbol{W}_1^{\mathrm{T}}\varphi_1(\bar{z}_1)+\varepsilon_1(\bar{z}_1)$，则 $|\varepsilon_1(\bar{z}_1)|\leqslant\varepsilon_1$，$\forall\bar{z}_1\in\Omega_{z_1}$. 于是式(11)可化为

$$\begin{aligned}\mathrm{L}V_{z_1}\leqslant&\frac{1}{\eta_1}+z_1^3[-k_1z_1-\tilde{\boldsymbol{W}}_1^{\mathrm{T}}\varphi_1(\bar{z}_1)+\\&\varepsilon_1(\bar{z}_1)]+\frac{1}{4\lambda_1^4}z_2^4.\end{aligned} \tag{13}$$

考虑如下 Lyapunov 函数

$$V_1=V_{z_1}+\frac{1}{2}\tilde{\boldsymbol{W}}_1^{\mathrm{T}}\boldsymbol{\Gamma}_1^{-1}\tilde{\boldsymbol{W}}_1, \tag{14}$$

取自适应律

$$\dot{\hat{\boldsymbol{W}}}_1=\boldsymbol{\Gamma}_1(z_1^3\varphi_1(\bar{z}_1)-\sigma_1\hat{\boldsymbol{W}}_1). \tag{15}$$

式中，$\boldsymbol{\Gamma}_1$ 为正定常矩阵，σ_1 为设计常数. 由式(3)、式(13)和式(15)可得

$$\mathrm{L}V_1\leqslant\frac{1}{\eta_1}-k_1z_1^4+\frac{1}{4\lambda_1^4}z_2^4+|z_1|^3\varepsilon_1-\sigma_1\tilde{\boldsymbol{W}}_1^{\mathrm{T}}\hat{\boldsymbol{W}}_1. \tag{16}$$

第 i 步($2\leqslant i\leqslant n-1$)，由系统(1)及式(5)可得

$$\begin{aligned}\mathrm{d}z_i=&[f_i(\bar{\boldsymbol{x}}_i)+g_i(\bar{\boldsymbol{x}}_i)x_{i+1}-\mathrm{L}\alpha_{i-1}]\mathrm{d}t+\\&\left(\boldsymbol{h}_i^{\mathrm{T}}(\bar{\boldsymbol{x}}_i)-\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\boldsymbol{h}_j^{\mathrm{T}}(\bar{\boldsymbol{x}}_j)\right)\mathrm{d}\boldsymbol{w}.\end{aligned} \tag{17}$$

式中，

$$\begin{aligned}\mathrm{L}\alpha_{i-1}=&\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}[f_j(\bar{\boldsymbol{x}}_j)+g_j(\bar{\boldsymbol{x}}_j)x_{j+1}]+\\&\frac{1}{2}\sum_{p,q=1}^{i-1}\frac{\partial^2\alpha_{i-1}}{\partial x_p\partial x_q}\boldsymbol{h}_p^{\mathrm{T}}(\bar{\boldsymbol{x}}_p)\boldsymbol{h}_q(\bar{\boldsymbol{x}}_q)+\beta_{i-1},\\\beta_{i-1}=&\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{W}_j}\dot{\hat{W}}_j+\frac{\partial\alpha_{i-1}}{\partial\boldsymbol{x}_{(i-1),\mathrm{d}}^{\mathrm{T}}}\dot{\boldsymbol{x}}_{(i-1),\mathrm{d}}.\end{aligned}$$

定义积分型 Lyapunov 函数

$$V_{z_i}=\int_0^{z_i}\frac{\sigma^3}{g_i(\bar{\boldsymbol{x}}_{i-1},\sigma+\alpha_{i-1})}\mathrm{d}\sigma \tag{18}$$

由式(3)可得

$$\begin{aligned}\mathrm{L}V_{z_i}=&\frac{z_i^3}{g_i(\bar{\boldsymbol{x}}_i)}(f_i(\bar{\boldsymbol{x}}_i)+g_i(\bar{\boldsymbol{x}}_i)x_{i+1}-\mathrm{L}\alpha_{i-1})+\\&\int_0^1\sum_{j=1}^{i-1}z_i^4\frac{\partial g_i^{-1}(\bar{\boldsymbol{x}}_{i-1},\rho_iz_i+\alpha_{i-1})}{\partial x_j}(f_j(\bar{\boldsymbol{x}}_j)+\\&g_j(\bar{\boldsymbol{x}}_j)x_{j+1})\mathrm{d}\rho_i+\left(\frac{z_i^3}{g_i(\bar{\boldsymbol{x}}_i)}-\right.\\&\left.\int_0^{z_i}\frac{3\sigma^2}{g_i(\bar{\boldsymbol{x}}_{i-1},\sigma+\alpha_{i-1})}\mathrm{d}\sigma\right)L\alpha_{i-1}+\\&\frac{3z_i^2g_i^{-1}(\bar{\boldsymbol{x}}_i)+z_i^3\frac{\partial g_i^{-1}(\bar{\boldsymbol{x}}_i)}{\partial z_i}}{2}\times\left(\boldsymbol{h}_i^{\mathrm{T}}(\bar{\boldsymbol{x}}_i)-\right.\\&\left.\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\boldsymbol{h}_j^{\mathrm{T}}(\bar{\boldsymbol{x}}_j)\right)\left(\boldsymbol{h}_i(\bar{\boldsymbol{x}}_i)-\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\boldsymbol{h}_j(\bar{\boldsymbol{x}}_j)\right)+\\&z_i^3\frac{\partial g_i^{-1}(\bar{\boldsymbol{x}}_i)}{\partial\alpha_{i-1}}\left(\boldsymbol{h}_i^{\mathrm{T}}(\bar{\boldsymbol{x}}_i)-\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\boldsymbol{h}_j^{\mathrm{T}}(\bar{\boldsymbol{x}}_j)\right)\cdot\\&\left(\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\boldsymbol{h}_j(\bar{\boldsymbol{x}}_j)\right)+z_i^3\sum_{p=1}^{i-1}\frac{\partial g_i^{-1}(\bar{\boldsymbol{x}}_i)}{\partial x_p}\left(\boldsymbol{h}_i^{\mathrm{T}}(\bar{\boldsymbol{x}}_i)-\right.\\&\left.\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\boldsymbol{h}_j^{\mathrm{T}}(\bar{\boldsymbol{x}}_j)\right)\boldsymbol{h}_p(\bar{\boldsymbol{x}}_p)+\frac{1}{2}\int_0^1\sum_{p,q=1}^{i-1}z_i^4\cdot\\&\frac{\partial^2g_i^{-1}(\bar{\boldsymbol{x}}_{i-1},\rho_iz_i+\alpha_{i-1})}{\partial x_p\,\partial x_q}\boldsymbol{h}_p^{\mathrm{T}}(\bar{\boldsymbol{x}}_p)\boldsymbol{h}_q(\bar{\boldsymbol{x}}_q)\mathrm{d}\rho_i+\\&\int_0^{z_i}\sum_{j=1}^{i-1}\sigma^3\frac{\partial^2g_i^{-1}(\bar{\boldsymbol{x}}_{i-1},\sigma+\alpha_{i-1})}{\partial x_j\,\partial\alpha_{i-1}}\mathrm{d}\sigma\boldsymbol{h}_j^{\mathrm{T}}(\bar{\boldsymbol{x}}_j)\cdot\\&\left(\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\boldsymbol{h}_j(\bar{\boldsymbol{x}}_j)\right)+\\&\frac{1}{2}\int_0^1z_i^4\frac{\partial^2g_i^{-1}(\bar{\boldsymbol{x}}_{i-1},\rho_iz_i+\alpha_{i-1})}{\partial\alpha_{i-1}^2}\mathrm{d}\rho_i\cdot\\&\left(\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\boldsymbol{h}_j^{\mathrm{T}}(\bar{\boldsymbol{x}}_j)\right)\left(\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\boldsymbol{h}_j(\bar{\boldsymbol{x}}_j)\right).\end{aligned} \tag{19}$$

由 Young's 不等式

$$\begin{aligned}&\frac{3z_i^2g_i^{-1}(\bar{\boldsymbol{x}}_i)}{2}\left(\boldsymbol{h}_i^{\mathrm{T}}(\bar{\boldsymbol{x}}_i)-\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\boldsymbol{h}_j^{\mathrm{T}}(\bar{\boldsymbol{x}}_j)\right)\cdot\\&\left(h_i^{\mathrm{T}}(\bar{\boldsymbol{x}}_i)-\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\boldsymbol{h}_j^{\mathrm{T}}(\bar{\boldsymbol{x}}_j)\right)^{\mathrm{T}}\leqslant\frac{1}{\eta_i}+\\&\frac{9\eta_i}{16g_i^2(\bar{\boldsymbol{x}}_i)}\left\|\boldsymbol{h}_i^{\mathrm{T}}(\bar{\boldsymbol{x}}_i)-\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\boldsymbol{h}_j^{\mathrm{T}}(\bar{\boldsymbol{x}}_j)\right\|^4z_i^4,\end{aligned} \tag{20}$$

$$z_i^3z_{i+1}\leqslant\frac{3}{4}\lambda_i^{\frac{4}{3}}z_i^4+\frac{1}{4\lambda_i^4}z_{i+1}^4. \tag{21}$$

将式(20)代入式(19)得

$$\mathrm{L}V_{z_i}\leqslant\frac{1}{\eta_i}+z_i^3(\alpha_i+\psi_i(\bar{z}_i))+\frac{1}{4\lambda_i^4}z_{i+1}^4, \tag{22}$$

$$
\begin{aligned}
\psi_i(\bar{z}_i) = &\frac{f_i(\bar{\boldsymbol{x}}_i)}{g_i(\bar{\boldsymbol{x}}_i)} + \frac{3}{4}\lambda_i^{\frac{4}{3}} z_i + \\
&\int_0^1 \sum_{j=1}^{i-1} z_i \frac{\partial g_i^{-1}(\bar{\boldsymbol{x}}_{i-1}, \rho_i z_i + \alpha_{i-1})}{\partial x_j}\big(f_j(\bar{\boldsymbol{x}}_j) + \\
&g_j(\bar{\boldsymbol{x}}_j) x_{j+1}\big)\mathrm{d}\rho_i - \mathrm{L}\alpha_{i-1}\int_0^1 \frac{3\rho_i^2}{g_i(\bar{\boldsymbol{x}}_{i-1}, \rho_i z_i + \alpha_{i-1})}\mathrm{d}\rho_i + \\
&\frac{9\eta_i}{16 g_i^2(\bar{\boldsymbol{x}}_i)}\left\| \boldsymbol{h}_i^{\mathrm{T}}(\bar{\boldsymbol{x}}_i) - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\boldsymbol{h}_j^{\mathrm{T}}(\bar{\boldsymbol{x}}_j)\right\|^4 z_i + \\
&\frac{1}{2}\frac{\partial g_i^{-1}(\bar{\boldsymbol{x}}_i)}{\partial z_i}\left\| \boldsymbol{h}_i^{\mathrm{T}}(\bar{\boldsymbol{x}}_i) - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\boldsymbol{h}_j^{\mathrm{T}}(\bar{\boldsymbol{x}}_j)\right\|^2 + \\
&\frac{\partial g_i^{-1}(\bar{\boldsymbol{x}}_i)}{\partial \alpha_{i-1}}\left(\boldsymbol{h}_i^{\mathrm{T}}(\bar{\boldsymbol{x}}_i) - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\boldsymbol{h}_j^{\mathrm{T}}(\bar{\boldsymbol{x}}_j)\right)\cdot \\
&\left(\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\boldsymbol{h}_j(\bar{\boldsymbol{x}}_j)\right) + \sum_{p=1}^{i-1}\frac{\partial g_i^{-1}(\bar{\boldsymbol{x}}_i)}{\partial x_p}\left(\boldsymbol{h}_i^{\mathrm{T}}(\bar{\boldsymbol{x}}_i) - \right. \\
&\left.\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\boldsymbol{h}_j^{\mathrm{T}}(\bar{\boldsymbol{x}}_j)\right)\boldsymbol{h}_p(\bar{\boldsymbol{x}}_p) + \frac{1}{2}\int_0^1 \sum_{p,q=1}^{i-1} z_i \cdot \\
&\frac{\partial^2 g_i^{-1}(\bar{\boldsymbol{x}}_{i-1}, \rho_i z_i + \alpha_{i-1})}{\partial x_p\,\partial x_q}\boldsymbol{h}_p^{\mathrm{T}}(\bar{\boldsymbol{x}}_p)\boldsymbol{h}_q(\bar{\boldsymbol{x}}_q)\mathrm{d}\rho_i + \\
&\int_0^1 \sum_{j=1}^{i-1} z_i \frac{\partial^2 g_i^{-1}(\bar{\boldsymbol{x}}_{i-1}, \rho_i z_i + \alpha_{i-1})}{\partial x_j\,\partial\alpha_{i-1}}\mathrm{d}\rho_i \boldsymbol{h}_j^{\mathrm{T}}(\bar{\boldsymbol{x}}_j)\cdot \\
&\left(\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\boldsymbol{h}_j(\bar{\boldsymbol{x}}_j)\right) + \frac{1}{2}\int_0^1 z_i \cdot \\
&\frac{\partial^2 g_i^{-1}(\bar{\boldsymbol{x}}_{i-1}, \rho_i z_i + \alpha_{i-1})}{\partial \alpha_{i-1}^2}\mathrm{d}\rho_i\left(\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\boldsymbol{h}_j^{\mathrm{T}}(\bar{\boldsymbol{x}}_j)\right)\cdot \\
&\left(\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\boldsymbol{h}_j(\bar{\boldsymbol{x}}_j)\right).
\end{aligned}
$$

式中，$\bar{z}_i = \left(\bar{\boldsymbol{x}}_i^{\mathrm{T}}, \alpha_{i-1}, \frac{\partial\alpha_{i-1}}{\partial x_1}, \cdots, \frac{\partial\alpha_{i-1}}{\partial x_{i-1}}, \frac{\partial^2\alpha_{i-1}}{\partial x_1^2}, \frac{\partial^2\alpha_{i-1}}{\partial x_1\,\partial x_2}, \cdots, \frac{\partial^2\alpha_{i-1}}{\partial x_{i-1}^2}, \beta_{i-1}\right)^{\mathrm{T}} \in \mathbf{R}^{(i^2+3i+2)/2}$.

取虚拟控制律

$$
\alpha_i = -k_i z_i - \frac{1}{4\lambda_{i-1}^4}z_i - \hat{\boldsymbol{W}}_i^{\mathrm{T}}\varphi_i(\bar{z}_i). \tag{23}
$$

式中：$k_i > 0$ 为设计常数；$\hat{\psi}_i(\bar{z}_i) = \hat{\boldsymbol{W}}_i^{\mathrm{T}}\varphi_i(\bar{z}_i)$ 为径向基函数神经网络在有界闭集 $\Omega_{\bar{z}_i}$ 上对 $\psi_i(\bar{z}_i)$ 的逼近；$\hat{\boldsymbol{W}}_i$ 为理想权向量 $\boldsymbol{W}_i$ 的估计；估计误差定义为 $\widetilde{\boldsymbol{W}}_i = \hat{\boldsymbol{W}}_i - \boldsymbol{W}_i$. 令 $\psi_i(\bar{z}_i) = \boldsymbol{W}_i^{\mathrm{T}}\varphi_i(\bar{z}_i) + \varepsilon_i(\bar{z}_i)$，则 $|\varepsilon_i(\bar{z}_i)| \leqslant \varepsilon_i, \forall \bar{z}_i \in \Omega_{\bar{z}_i}$. 于是式(22)可化为

$$
\mathrm{L}V_{z_i} \leqslant \frac{1}{\eta_i} + z_i^3\left(-k_i z_i - \widetilde{\boldsymbol{W}}_i^{\mathrm{T}}\varphi_i(\bar{z}_i) + \varepsilon_i(\bar{z}_i)\right) + \frac{1}{4\lambda_i^4}z_{i+1}^4 \tag{24}
$$

考虑如下 Lyapunov 函数

$$
V_i = V_{i-1} + V_{z_i} + \frac{1}{2}\widetilde{\boldsymbol{W}}_i^{\mathrm{T}}\boldsymbol{\Gamma}_i^{-1}\widetilde{\boldsymbol{W}}_i, \tag{25}
$$

取自适应律

$$
\dot{\hat{\boldsymbol{W}}}_i = \boldsymbol{\Gamma}_i\left(z_i^3\varphi_i(\bar{z}_i) - \sigma_i\hat{\boldsymbol{W}}_i\right). \tag{26}
$$

式中，$\boldsymbol{\Gamma}_i$ 为正定常矩阵，σ_i 为设计常数. 由式(3)、式(24)和式(26)可得

$$
\mathrm{L}V_i \leqslant \sum_{j=1}^{i}\frac{1}{\eta_j} - \sum_{j=1}^{i}\left(-k_j z_j^4 + \frac{1}{4\lambda_i^4}z_{i+1}^4\right) + \sum_{j=1}^{i}|z_j|^3\varepsilon_j - \sum_{j=1}^{i}\sigma_j\widetilde{\boldsymbol{W}}_j^{\mathrm{T}}\hat{\boldsymbol{W}}_j. \tag{27}
$$

当 $i = n$ 时，$z_{n+1} = 0$，实际的控制器 u 及参数自适应律可构造如下：

$$
u = -k_n z_n - \frac{1}{4\lambda_{n-1}^4}z_n - \hat{\boldsymbol{W}}_n^{\mathrm{T}}\varphi_n(\bar{z}_n), \tag{28}
$$

$$
\dot{\hat{\boldsymbol{W}}}_n = \boldsymbol{\Gamma}_n\left(z_n^3\varphi_n(\bar{z}_n) - \sigma_n\hat{\boldsymbol{W}}_n\right). \tag{29}
$$

相应地，由式(28)、式(29)可得

$$
\mathrm{L}V_n \leqslant \sum_{j=1}^{n}\frac{1}{\eta_j} - \sum_{j=1}^{n}k_j z_j^4 + \sum_{j=1}^{n}|z_j|^3\varepsilon_j - \sum_{j=1}^{n}\sigma_j\widetilde{\boldsymbol{W}}_j^{\mathrm{T}}\hat{\boldsymbol{W}}_j. \tag{30}
$$

由 Young's 不等式

$$
-\sigma_j\widetilde{\boldsymbol{W}}_j^{\mathrm{T}}\hat{\boldsymbol{W}}_j = -\sigma_j\widetilde{\boldsymbol{W}}_j^{\mathrm{T}}(\widetilde{\boldsymbol{W}}_j + \boldsymbol{W}_j) \leqslant -\frac{\sigma_j\|\widetilde{\boldsymbol{W}}_j\|^2}{2} + \frac{\sigma_j\|\boldsymbol{W}_j\|^2}{2},
$$

$$
|z_j|^3\varepsilon_j \leqslant \frac{3}{4}k_j z_j^4 + \frac{1}{4k_j^3}\varepsilon_j^4.
$$

故式(30)可化为

$$
\mathrm{L}V_n \leqslant \sum_{j=1}^{n}\frac{1}{\eta_j} - \frac{1}{4}\sum_{j=1}^{n}k_j z_j^4 - \sum_{j=1}^{n}\frac{\sigma_j\|\widetilde{\boldsymbol{W}}_j\|^2}{2} + \sum_{j=1}^{n}\frac{\sigma_j\|\boldsymbol{W}_j\|^2}{2} + \sum_{j=1}^{n}\frac{1}{4k_j^3}\varepsilon_j^4 \leqslant -cV_n + \mu. \tag{31}
$$

式中：$c = \min\limits_{i=1,\cdots,n}\left\{k_i g_{i0}, \frac{\sigma_i}{\lambda_{\max}(\boldsymbol{\Gamma}_i^{-1})}\right\}$，$\lambda_{\max}(\boldsymbol{\Gamma}_i^{-1})$ 为矩阵 $\boldsymbol{\Gamma}_i^{-1}$ 的最大特征值；$\mu = \sum_{j=1}^{n}\frac{1}{\eta_j} + \sum_{j=1}^{n}\frac{\sigma_j\|\boldsymbol{W}_j\|^2}{2} + \sum_{j=1}^{n}\frac{1}{4k_j^3}\varepsilon_j^4$. 由引理 1 可知，系统存在唯一解，且解过程是依概率有界的.

3 仿真结果

考虑如下随机非线性系统：

$$
\begin{cases}
\mathrm{d}x_1 = \left[x_1\mathrm{e}^{-0.5x_1} + (1 + x_1^2)x_2\right]\mathrm{d}t + 0.1\sin x_1\mathrm{d}w, \\
\mathrm{d}x_2 = \left[x_1 x_2^2 + (3 + \cos(x_1 x_2))u\right]\mathrm{d}t + x_1\mathrm{e}^{x_2}\mathrm{d}w, \\
y = x_1.
\end{cases}
$$

控制目标是使输出 y 跟踪期望信号：

$$
y_{\mathrm{d}} = 0.5\sin t + \sin 0.5t.
$$

选择虚拟控制律、控制律和参数自适应律如下：

$$\alpha_1 = -k_1 z_1 - \hat{W}_1^{\mathrm{T}} \varphi_1(\bar{z}_1),$$

$$u = -k_2 z_2 - \frac{1}{4\lambda_1^4} z_2 - \hat{W}_2^{\mathrm{T}} \varphi_2(\bar{z}_2),$$

$$\dot{\hat{W}}_i = \boldsymbol{\Gamma}_i (z_i^3 \varphi_i(\bar{z}_i) - \sigma_i \hat{W}_i), \quad i = 1, 2.$$

式中，$z_1 = x_1 - y_{\mathrm{d}}$，$z_2 = x_2 - \alpha_1$，$\bar{z}_1 = (x_1, y_{\mathrm{d}}, \dot{y}_{\mathrm{d}})^{\mathrm{T}}$，$\bar{z}_2 = \left(x_1, x_2, \alpha_1, \frac{\partial \alpha_1}{\partial x_1}, \frac{\partial^2 \alpha_1}{\partial x_1^2}, \beta_1\right)^{\mathrm{T}}$，$\beta_1 = \frac{\partial \alpha_1}{\partial y_{\mathrm{d}}} \dot{y}_{\mathrm{d}} + \frac{\partial \alpha_1}{\partial \dot{y}_{\mathrm{d}}} \ddot{y}_{\mathrm{d}} + \frac{\partial \alpha_1}{\partial \hat{w}_1} \dot{\hat{w}}_1$，神经网络节点数 $l_1 = 50$，$l_2 = 60$，基函数的宽度 $\mu_{1i} = 0.02(i - 25)$，$i = 1, 2, \cdots, 50$，$\mu_{2i} = 0.03(i - 30)$，$i = 1, 2, \cdots, 60$，中心 $\eta_{1i} = \eta_{2i} = 1$. 初始条件为 $(x_1(0), x_2(0))^{\mathrm{T}} = (0.1, -0.2)^{\mathrm{T}}$，$(\hat{w}_1^{\mathrm{T}}(0), \hat{w}_2^{\mathrm{T}}(0))^{\mathrm{T}} = (0, 0)^{\mathrm{T}}$，设计参数为 $k_1 = k_2 = 40$，$\Gamma_1 = \Gamma_2 = \mathrm{diag}(10, 10)$，$\sigma_1 = \sigma_2 = 0.01$. 仿真结果如图 1 和图 2 所示.

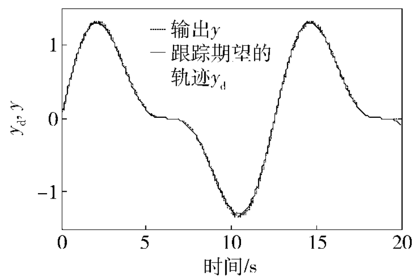

图 1　输出 y 跟踪期望的轨迹 y_{d}

Fig. 1　Output y following the desired trajectory y_{d}

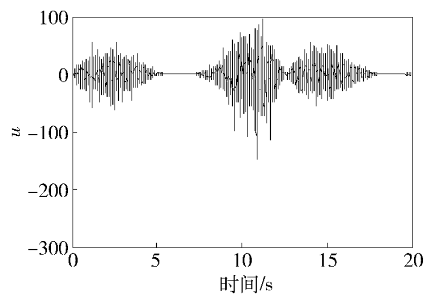

图 2　控制信号 u

Fig. 2　Control signal u

4　结论

讨论了一类随机严格反馈非线性系统的自适应跟踪控制问题. 通过引入积分型 Lyapunov 函数，并基于后推设计、Young's 不等式以及径向基函数神经网络的逼近性质，提出了一种自适应神经网络跟踪控制策略. 根据 Lyapunov 方法，在后推设计的每一步确定了可调参数的自适应律. 理论分析证明了闭环系统的所有信号依概率有界.

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